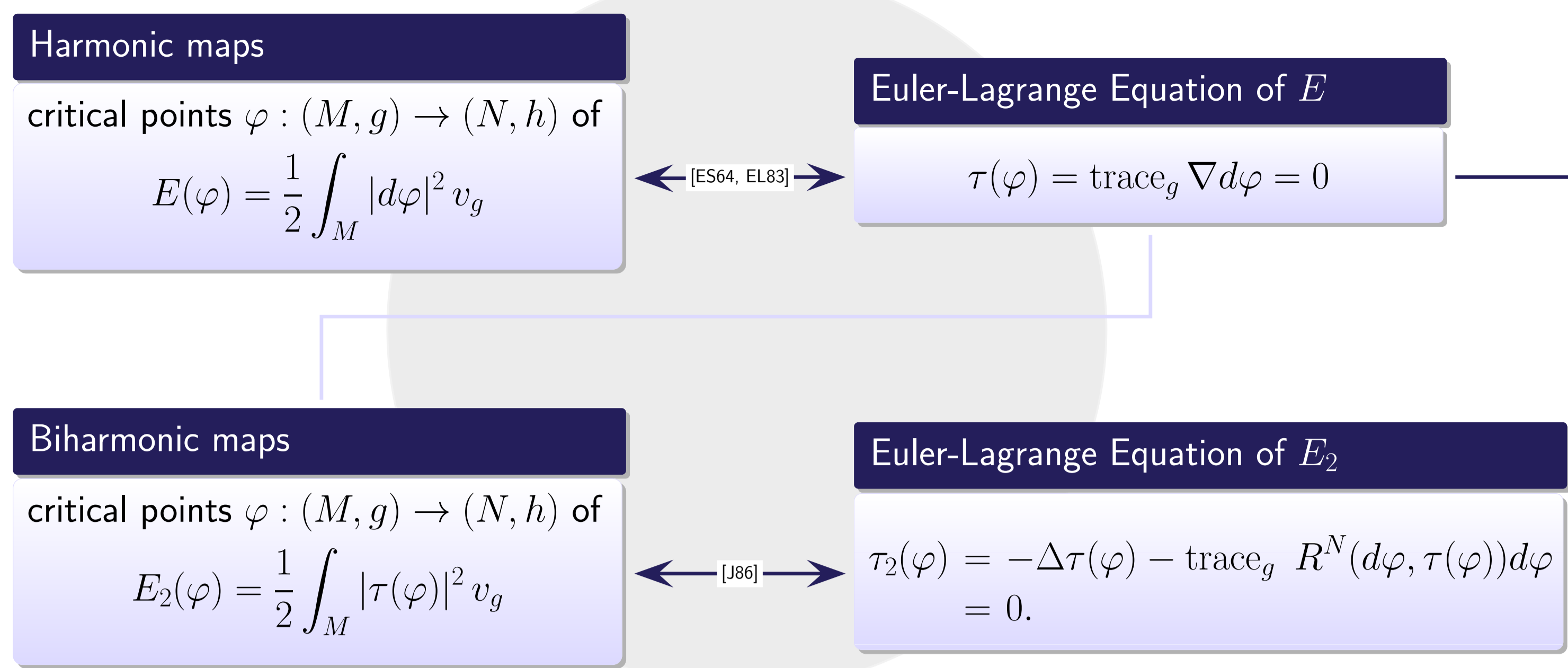
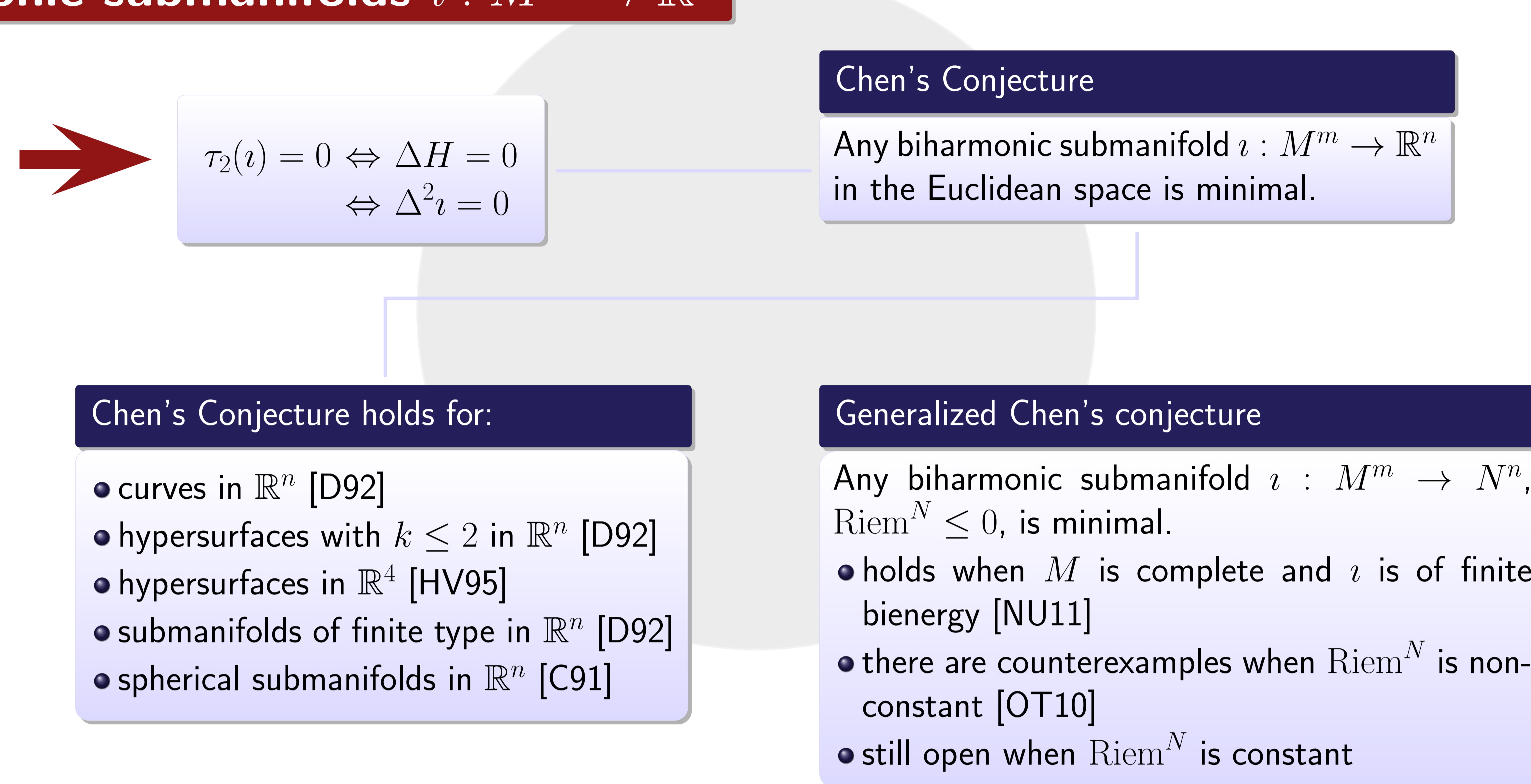


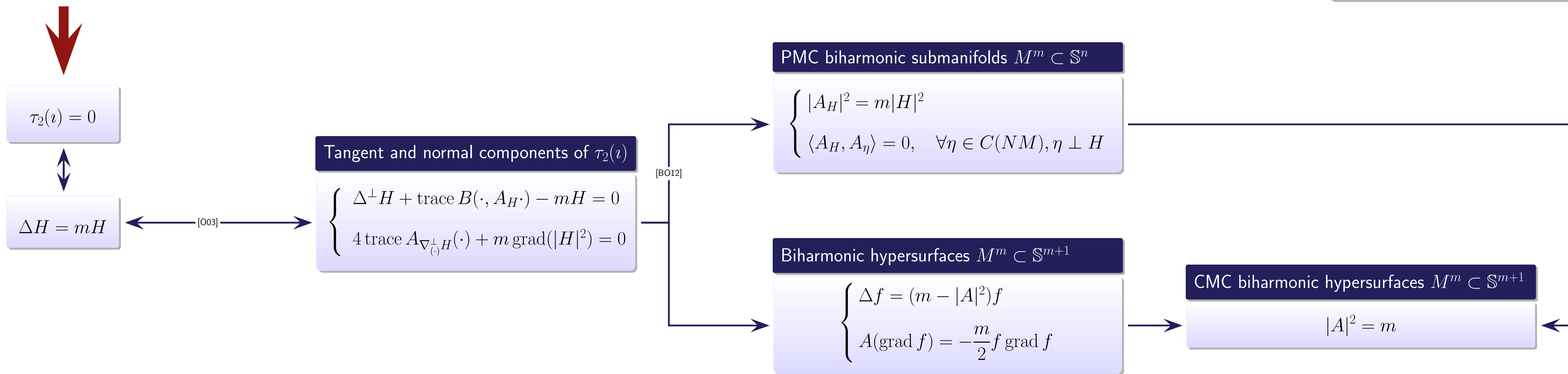
Definitions



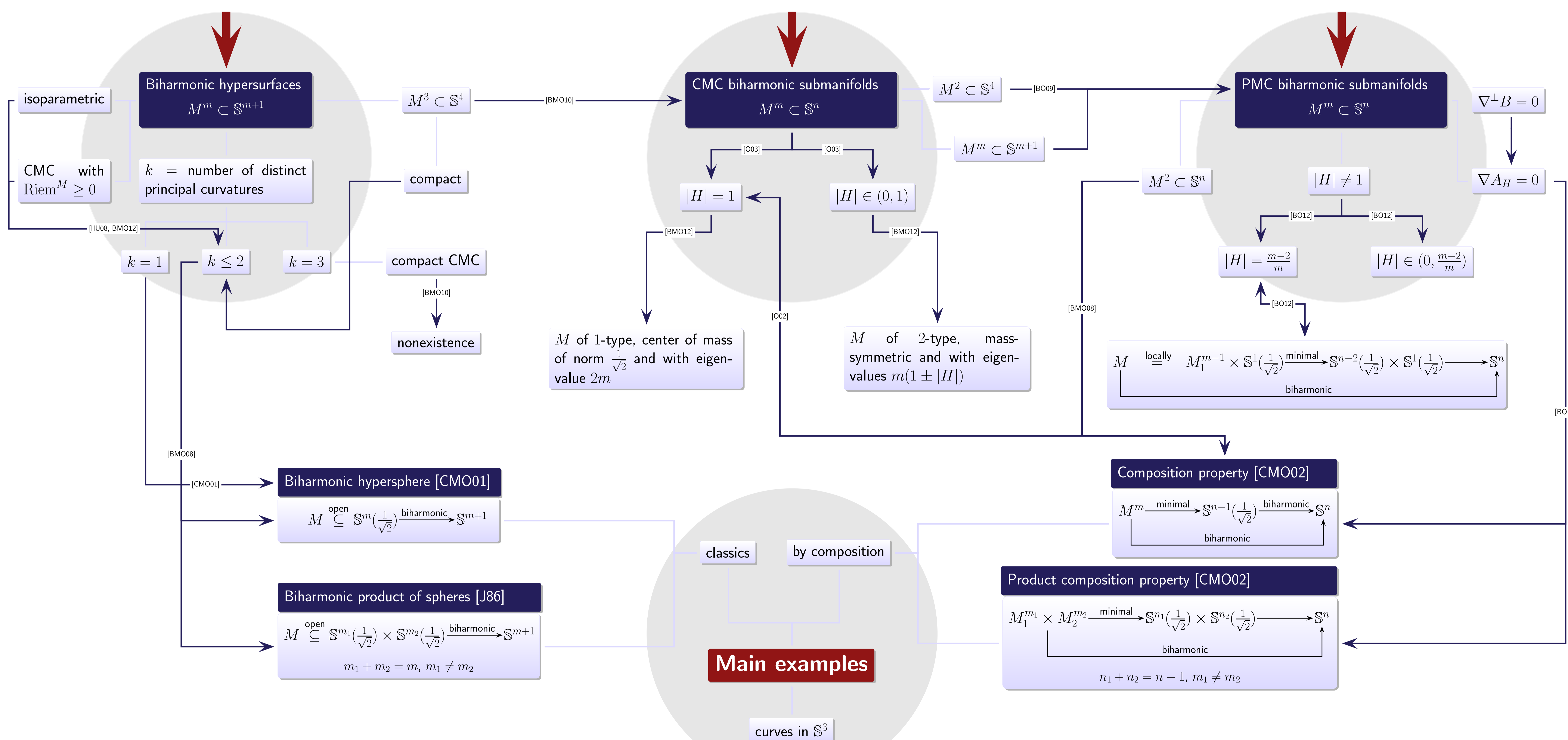
Bi-harmonic submanifolds $\iota : M^m \rightarrow \mathbb{R}^n$



Bi-harmonic submanifolds $\iota : M^m \rightarrow S^n$ - characterization results



Bi-harmonic submanifolds $\iota : M^m \rightarrow S^n$ - main examples and classification results



OPEN PROBLEMS

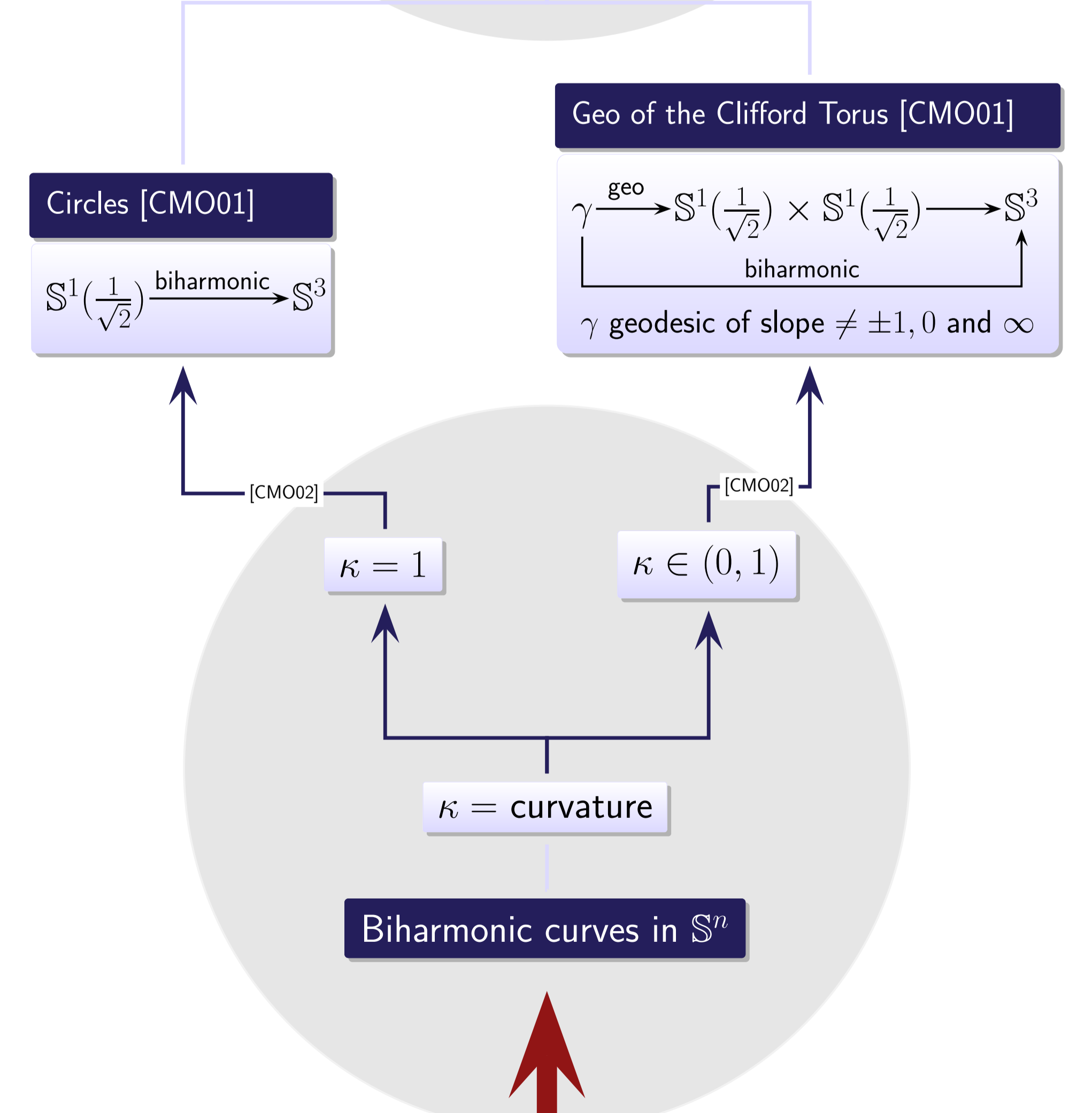
Conjecture 1. The only proper-biharmonic hypersurfaces in S^{m+1} are the open parts of hyperspheres $S^m(1/\sqrt{2})$ or of the standard products of spheres $S^{m_1}(1/\sqrt{2}) \times S^{m_2}(1/\sqrt{2})$, $m_1 + m_2 = m, m_1 \neq m_2$.

Statements equivalent to Conjecture 1.

- A proper-biharmonic hypersurface in S^{m+1} has at most two principal curvatures everywhere.
- A proper-biharmonic hypersurface in S^{m+1} is parallel.
- A proper-biharmonic hypersurface in S^{m+1} is CMC and has non-negative sectional curvature.
- A proper-biharmonic hypersurface in S^{m+1} is isoparametric.

Conjecture 2. The proper-biharmonic hypersurfaces in S^{m+1} are CMC.

Problem. Find a PMC proper-biharmonic submanifold M^m in S^n such that A_H is not parallel.



NOTATIONS AND CONVENTIONS

This poster illustrates the known results on the classification of bi-harmonic submanifolds in S^n .

- bi-harmonic submanifold = bi-harmonic isometric immersion
- proper-biharmonic = bi-harmonic non-harmonic
- $\Delta^2 = \Delta \circ \Delta$
- $\Delta = -\text{trace}_g (\nabla^\perp \nabla^\perp - \nabla_{\mathbb{S}^n}^2)$ is the rough Laplacian.
- $R^N(X, Y) = [\nabla_X, \nabla_Y] - \nabla_{[X, Y]}$ is the curvature on N .
- B is the second fundamental form of M in S^n .
- H is the mean curvature vector field of M^m in S^n .
- A_H is the Weingarten operator of M^m associated to H .
- A is the shape operator of M^m in S^{m+1} and $f = \text{trace } A/m$.
- k is the number of distinct principal curvatures of a hypersurface.

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