Existence result for differential inclusion with $p(x)$-Laplacian

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Abstract

The hemivariational inequalities have been introduced and studied since 1981 by P.D.Panagiotopoulos as the mathematical models of many problems coming from mechanics, engineering and economics. They are variational formulations of problems with nonconvex energy functions and they are derived with the help of the generalized subdifferential in the sense of Clarke. Hemivariational inequalities are an effective tool to treat problems with nonmonotonicity and multivaluedness.

Let $\Omega \subseteq \mathbb{R}^N$ be a bounded domain with a $\mathcal{C}^2$-boundary $\partial\Omega$ and $N > 2$. We study the following nonlinear elliptic differential inclusion with $p(x)$-Laplacian

$$\begin{cases} -\Delta_{p(x)}u - \lambda|u(x)|^{p(x)-2}u(x) \in \partial j(x, u(x)) \text{a.e. on } \Omega, \\ u = 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $p : \overline{\Omega} \to \mathbb{R}$ is a continuous function satisfying $1 < p^- := \inf_{x\in\Omega} p(x) \leqslant p(x) \leqslant p^+ := \sup_{x\in\Omega} p(x) < N < \infty$, $p^+ \leqslant \hat{p}^* := \frac{Np^-}{N-p^-}$ and $j(x,t)$ is a function locally Lipschitz in the t-variable and measurable in x-variable. By $\partial j(x,t)$ we denote the subdifferential with respect to the t-variable in the sense of Clarke. The operator $\Delta_{p(x)}u := \operatorname{div}\big(|\nabla u(x)|^{p(x)-2}\nabla u(x)\big)$ is the so-called $p(x)$-Laplacian, which becomes p-Laplacian when $p(x) \equiv p$.

In our problem appears λ, for which we assume that $\lambda < \frac{p^-}{p^+}\lambda_*$, where λ_* is introduced by the following Rayleigh quotient

$$\lambda_* = \inf_{u \in W_0^{1,p(x)}(\Omega)\setminus\{0\}} \frac{\int_\Omega |\nabla u(x)|^{p(x)} dx}{\int_\Omega |u(x)|^{p(x)} dx}.$$

In such kind of problems we usually consider different types of sufficient conditions for the existence of solutions. In the literature we can meet Landesman-Lazer condition, Ambrosetti-Rabinowitz or Tang condition as well as some other. These conditions are usually not necessary. Problem of finding the possible widest class of nonlinearities for which nonlinear Dirichlet boundary value problem with $p(x)$-Laplacian has a solution is still open.

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