Structure of p-complements of finite groups and p-regular class sizes

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Abstract

The structure of a finite group G is strongly controlled by its set of conjugacy class sizes. For any prime p, it has been put forward that certain properties on the sizes of the p-regular conjugacy classes of G, that is, those classes of p'-order elements, also affect the p-structure of G. For instance, if the set of p-regular class sizes of G is $\{1, m\}$ for some integer m, then the p-complements of G are nilpotent. We provide certain extensions for p-solvable groups of the celebrated Itô's theorem on groups with two class sizes, which asserts that any group having two class sizes is nilpotent. More precisely, we determine the structure of the p-complements of a p-solvable group G, when the set of p-regular class sizes of G is either $\{1, p^a, mp^a\}$ or $\{1, m, mp^a\}$, where m is any integer coprime to p.

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