Pluricomplex Green function via Bernstein functions and Markov constants

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Abstract

The pluricomplex Green function V_E of a nonpluripolar compact set $E \subset \mathbb{C}^N$ is closely related to polynomials in view of the known formulas

$$V_E(z) = \log \lim_{n \to \infty} \sup \left\{ \frac{|P(z)|^{1/n}}{||P||_E^{1/n}} : P : \mathbb{C}^N \to \mathbb{C} \text{ polynomial of degree } n \right\}$$

where $|| \cdot ||_E$ is the maximum norm on E. We show that V_E is also connected with derivatives of polynomials. For this purpose, we consider Bernstein functions and Markov constants associated with the classical Bernstein and Markov inequalities. An extremal-like function $\varphi_E^{[\alpha]}$ (defined in terms of Bernstein functions for the derivatives D^α of polynomials) is plurisubharmonic in $\mathbb{C}^N$ and very close to V_E. Moreover, for any multi-index α, the function $\log \varphi_E^{[\alpha]}$ is equal to V_E for a large class of sets, e.g. for Markov sets and for all compacts with continuous pluricomplex Green function.

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