The Cremona Group

Serge Cantat serge.cantat@univ-rennes1.fr
École Normale Supérieure,
45 rue d'Ulm, Paris

Abstract

The Cremona group is the group of all birational transformations of the plane; using affine coordinates (x, y), each element f of this group can be expressed in the form $f(x, y) = (p(x, y), q(x, y))$ where p and q are rational fractions in x and y. This group is infinite dimensional, and contains elements with rich dynamical behaviors. In this respect, it looks like groups of diffeomorphisms of compact manifolds. On the other hand, it shares interesting properties with smaller groups, like linear groups or Lie groups. I shall describe the basic features of the Cremona group, and explain how algebraic geometry, dynamical systems, and geometric group theory can be combined simultaneously to describe the structure of this group of transformations.

AMS Classification: 14E07, 20E32, 32M05, 37F99.