Anomalous behaviour of solutions to aggregation equation in bounded and unbounded domain

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Abstract

My poster will consists of two parts. First, we consider the so-called aggregation equation on unbounded and one-dimensional domain, namely, the following initial value problem

$$u_t = \varepsilon u_{xx} + \left(u\ K' * u\right)_x \quad \text{for } x \in \mathbb{R},\ t > 0, \tag{1}$$

$$u(x,0) = u_0(x) \quad \text{for } x \in \mathbb{R}, \tag{2}$$

where the initial datum $u_0 \in L^1(\mathbb{R})$ is nonnegative and $\varepsilon \geq 0$.
Notice, that the famous parabolic-eliptic system of chemotaxis (Keller-Segel model)

$$u_t = u_{xx} - (uv_x)_x, \quad -v_{xx} + av = u, \qquad x \in \Omega,\ t > 0 \tag{3}$$

can be written as equation (1). Indeed, if we put $K(x) = -\frac{1}{2}e^{-|x|}$ into the (1), which is the fundamental solution of the operator $\partial_x^2 + \mathrm{Id}$, one can rewrite the second equation in (3) as $v = K * v$.
We show that, under some strong assumptions on K, the asymptotic behaviour, when $t \to \infty$, of solutions to problem (1)-(2), describe the function given by the explicit formula.

In the second part we study the following initial boundary value problem

$$u_t = \nabla \cdot (\nabla u - u\nabla \mathcal{K}(u)) \quad \text{for } x \in \Omega \subset \mathbb{R}^d,\ t > 0, \tag{4}$$

$$\frac{\partial u}{\partial n} = 0 \quad \text{for } x \in \partial\Omega,\ t > 0 \tag{5}$$

$$u(x,0) = u_0(x) \quad \text{for } x \in \Omega, \tag{6}$$

where the initial datum $u_0 \in L^1(\Omega)$ is nonnegative and the operator $\mathcal{K}$ depends linearly on u via the following integral formula

$$\mathcal{K}(u)(x,t) = \int_\Omega K(x,y)u(y,t)dy \tag{7}$$

for a certain function $K = K(x,y)$ which we call as an aggregation kernel. In this part we consider quite weak assumptions on the function K.
Notice, that problem (4)-(6) is rather rarely studied. Most of results on aggregation equation refer to the whole space $\mathbb{R}^d$. Therefore, at the beginning, we show theorems concerning existence of solutions. Later on, we deliver sufficient conditions for stability and instability of constant stationary solutions to (4)-(6).

References

[1] R. Celiński, *Asymptotic behaviour in one dimensional model of interacting particles*, Nonlinear Analysis, 75 (2012), 1972-1979.

[2] R. Celiński, *Stability of solutions to aggregation equation in a bounded domains*, submitted.