The p-adic Langlands program

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Abstract

The group $G_{\mathbf{Q}}$ of automorphisms of the field $\overline{\mathbf{Q}}$ of algebraic numbers is still a very mysterious object despite continuous efforts from number theorists for more than 150 years. One way to understand this group is to consider its representations, and Langlands has proposed a program in 1967 which seeks to describe the representations of $G_{\mathbf{Q}}$ coming from geometry in terms of harmonic analysis on the various completions of $\mathbf{Q}$ (i.e. $\mathbf{R}$ and the fields of p-adic numbers $\mathbf{Q}_p$ for prime numbers p). The last decade has seen the apparition of a p-adic avatar of Langlands's program following Wiles's proof of Fermat's last theorem and the ensuing proof of the Taniyama-Weil conjecture (a special but crucial case of Langlands's correspondence) by Breuil-Conrad-Diamond-Taylor. This p-adic program is still in infancy, but the case of representations of dimension 2 is rather well understood by now. Compared with the classical case, the p-adic picture has some very nice features that I will emphasise in my talk.