Extremal Laurent Polynomials and Fano manifolds

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Abstract

Given a Laurent polynomial f, I explain how to construct the Picard–Fuchs differential operator L_f and its natural solution, the principal period π_f. By definition, f is extremal if L_f has smallest possible ramification. I introduce two classes of Laurent polynomials in 3 variables that are conjectured to be extremal: Minkowski polynomials and polynomials with boundary motive of Hodge–Tate type. I briefly summarize certain facts about the quantum cohomology of a Fano manifold X, introducing the regularized quantum differential operator $\widehat{Q}_X$ and power series solution $\widehat{J}_X$. Conjecturally, $\widehat{Q}_X$ is of small (often minimal) ramification. A Fano manifold X is mirror-dual to a Laurent polynomial f if $\widehat{Q}_X = L_f$. I demonstrate how to derive the classification of Fano 3-folds (Iskovskikh, Mori–Mukai) from the classification of 3-variable Minkowski polynomials. In conclusion, I give a status report on a program to classify Fano 4-folds using these ideas.

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