Dissipative solutions of the Euler equations

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Abstract

The incompressible Euler equations were derived more than 250 years ago by Euler to describe the motion of an inviscid incompressible fluid. It is known since the pioneering works of Scheffer and Shnirelman that there are nontrivial distributional solutions to these equations which are compactly supported in space and time. If they were to model the motion of a real fluid, we would see it suddenly start moving after staying at rest for a while, without any action by an external force. A celebrated theorem by Nash and Kuiper shows the existence of C^1 isometric embeddings of a fixed flat rectangle in arbitrarily small balls of the threedimensional space. You should therefore be able to put a fairly large piece of paper in a pocket of your jacket without folding it or crumpling it.

In a first joint work with László Székelyhidi we pointed out that these two counterintuitive facts share many similarities. This has become even more apparent in a more recent result of ours, which proves the existence of continuous solutions which dissipate the kinetic energy. Our theorem might be regarded as a first step towards a conjecture of Lars Onsager, which in his 1949 paper about the theory of turbulence asserted the existence of dissipative Hölder solutions.

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