The FD-method for solving nonlinear Sturm-Liouville problems with distribution potentials

Denys Dragunov dragunovdenis@gmail.com
Institute of Mathematics, National Academy of Sciences, 3 Tereschenkivska St., 01601 Kyiv, Ukraine

Abstract

The poster is devoted to a numerically-analytical method for solving nonlinear Sturm-Liouville problems of the form

$$u''(x) + [\lambda - q(x) - \beta\delta(x-\alpha) - N(u(x))]\,u(x) = 0,\ x \in (0,1)\backslash\{\alpha\},$$

$u(0) = u(1) = 0, u'(0) = 1,$ where $\alpha \in (0,1), \beta \in R,\ q(x) \in L^1(0,1),$ function $N(u)$ is holomorphic on $(-\infty,+\infty)$ and $\delta(x)$ denotes the Dirac delta function. Such problems are of great interest in quantum mechanics. The method (FD-method) can be viewed as a symbiosis of the coefficient approximation method (Pruss method) with the homotopy technique. The method allows one to approximate the eigenfunction $u_n(x)$ and corresponding eigenvalue λ_n in the form of truncated series $u_n^{[m]}(x) = \sum_{i=0}^{m} u_n^{(i)}(x)$ and $\lambda_n^{[m]} = \sum_{i=0}^{m} \lambda_n^{(i)}$ respectively. It is proved that under certain conditions the method converges superexponentially and the convergence rate increases as the index n of the trial eigenpair increases, that is, $\|u_n^{[m]}(x) - u_n(x)\|_\infty \leq \frac{c_1}{m}(\frac{c}{n})^{m+1}$, $|\lambda_n^{[m]} - \lambda_n(x)| \leq \frac{c_1}{m}(\frac{c}{n})^{m}$, for some real c, c_1. The algorithm of the method can be efficiently implemented either analytically or numerically.

AMS Classification: 65L15.