Almost disjointness and its generalizations

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Abstract

Two subsets of the set of the natural numbers $\mathbb{N}$ are called almost disjoint, if their intersection is finite. A family $\mathcal{F} \subseteq \mathcal{P}(\mathbb{N})$ is called almost disjoint (in short: AD) if any two sets in the family are almost disjoint. An AD family $\mathcal{F}$ is maximal (in short: MAD family) if it cannot be extended by any infinite subset of $\mathbb{N}$ i.e. for every infinite set $A \subseteq \mathbb{N}$ there exists a set $F \in \mathcal{F}$ such that the intersection $A \cap F$ is infinite.

The minimal cardinality of a MAD family of subsets of $\mathbb{N}$ is one of the standard cardinal characteristics of the continuum, which is denoted by $\mathfrak{a}$. The almost disjointness number $\mathfrak{a}$ is uncountable and does not exceed the cardinality of the continuum. Its exact value is independent of ZFC set theory and it varies in different models of set theory.

Recently, some generalizations of almost disjointness with respect to a given ideal $\mathcal{I}$ were introduced and studied. In this setting two sets are said to be $\mathcal{I}$-almost disjoint if their intersection belongs to the ideal $\mathcal{I}$. One can introduce a new cardinal characteristics $\mathfrak{a}(\mathcal{I})$ — the minimal cardinality of a maximal $\mathcal{I}$-almost disjoint family.

We investigate $\mathfrak{a}(\mathcal{I})$ in the case $\mathcal{I} = \mathcal{W}$, where $\mathcal{W}$ is the van der Waerden ideal which consists of the sets which do not contain arbitrarily long arithmetic progressions. We prove, in particular, that $\mathfrak{a}(\mathcal{W})$ is less or equal to $\mathfrak{a}$. Some other cardinal characteristics related to $\mathcal{W}$ (or other ideals) may be considered as well if space permits.

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