

Sum of convex compacta as attractor of a hyperbolic IFS

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Abstract

M. Barnsley has introduced the idea of Iterated Function System with condensation, which means a hyperbolic IFS, accompanied by a constant compact-valued multi-function (condensation). This idea conducted to new fractals as attractors of IFS's, mostly related to Cantor sets. However, the computer simulation of such an IFS creates more problems than for a hyperbolic IFS.

S. Crovisier and M. Rams have constructed a compact in R^3 , which can not serve as attractor of any hyperbolic IFS. Some examples are studied also by P. F. Duvall, L. S. Husch, M. Kwieciński and others.

In this connection some problems arise:

Which compacta can serve as attractors of hyperbolic IFS's (or with condensation)? Given a compact, how to estimate the minimal number of functions of IFS needed to obtain this compact as attractor?

The last question is related also to the Borsuk conjecture.

We prove that any finite sum (reunion) of convex compacta in R^n can be represented as attractor of a hyperbolic IFS. We show also that for any IFS with such a condensation set, there exists a hyperbolic IFS, having the same attractor.

As a consequence, we construct the well-known fractal Pythagoras tree (attractor of an IFS with condensation) by CAS Mathematica, using only five contractions.

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