The Matrix Logarithm: from Theory to Computation

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Abstract

We describe the theory of the matrix logarithm and present some new algorithms for computing it and its condition number. The matrix logarithm arises in a number of applications (e.g., Markov models, optics, inverse problems) and we begin by outlining some of them. We describe some key properties of the set of matrix logarithms, giving particular attention to non-primary logarithms, whose existence is what makes the embeddability problem for Markov chains so hard. We present a new inverse scaling and squaring algorithm for computing the matrix logarithm, which employs a matrix analogue of the method Briggs used in the 17th century to produce his tables of logarithms. Numerical evaluation of the Fréchet derivative and estimation of the condition number, important for optimization and measuring sensitivity of the logarithm, will also be discussed.

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