

# ENERGY-MINIMAL DIFFEOMORPHISMS BETWEEN DOUBLY CONNECTED RIEMANN SURFACES

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ABSTRACT. Let  $N = (\Omega, \sigma)$  and  $M = (\Omega^*, \rho)$  be doubly connected Riemann surfaces and assume that  $\rho$  is a smooth metric with bounded Gauss curvature  $\mathcal{K}$  and finite area. The paper establishes the existence of homeomorphisms between  $\Omega$  and  $\Omega^*$  that minimize the Dirichlet energy. *Among all homeomorphisms  $f: \Omega \xrightarrow{\text{ontq}} \Omega^*$  between doubly connected domains such that  $\text{Mod } \Omega \leq \text{Mod } \Omega^*$  there exists, unique up to conformal automorphisms of  $\Omega$ , an energy-minimal diffeomorphism which is a harmonic diffeomorphism. The results improve and extend some recent results of Iwaniec, Koh, Kovalev and Onninen (Inven. Math. (2011)) and can be considered as a variation of results of Iwaniec, Kovalev, Onninen (Proc. Roy. Soc. Edinburgh Sect. A (2011)) where the authors considered doubly connected domains in the complex plane w.r. to Euclidean metric.*

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