Periodic Laplace-Beltrami operator with preassigned spectral gaps

Andrii Khrabustovskyi andry9@ukr.net
B.Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, Lenin avenue 47, 61103 Kharkiv, Ukraine

Abstract

It is known (E.L. Green (1997), O. Post (2003)) that for an arbitrary $m \in \mathbb{N}$ one can construct a periodic non-compact Riemannian manifold M with at least m gaps in the spectrum of the corresponding Laplace-Beltrami operator $-\Delta_M$. Our goal is not only to produce a new type of periodic manifolds with spectral gaps but also to control the edges of these gaps. The main result is the following

Theorem. `[A. Khrabustovskyi, J.Differ.Equ. 252(2012)]` Let $L > 0$ be an arbitrary number, let $(\alpha_j, \beta_j) \subset [0, L]$ $(j = 1, \ldots, m)$ be arbitrary pairwise disjoint intervals. Let $n \in \mathbb{N} \setminus \{1\}$. Then one can construct the family $\{M^\varepsilon\}_\varepsilon$ of periodic n-dimensional manifolds such that the spectrum of the operator $-\Delta_{M^\varepsilon}$ has the following form in $[0, L]$:

$$\sigma(-\Delta_{M^\varepsilon}) \cap [0, L] = [0, L] \setminus \left(\cup_{j=1}^{m} (\alpha_j^\varepsilon, \beta_j^\varepsilon)\right)$$

where the intervals $(\alpha_j^\varepsilon, \beta_j^\varepsilon)$ satisfy

$$\forall j = 1, \ldots, m : \lim_{\varepsilon \to 0} \alpha_j^\varepsilon = \alpha_j, \lim_{\varepsilon \to 0} \beta_j^\varepsilon = \beta_j$$

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