Almost complex and symplectic circle actions with a few fixed points.

According to paper [4], manifolds admitting symplectic circle actions with exactly two fixed points may exist only in dimensions 2 and 6, and no example of such action is known in dimension 6. We extend this result to almost complex circle actions using a simple trick with Hirzebruch χ_y-genera ([2]) and show that there is an infinite family of six-dimensional S^1-manifolds with S^1-invariant almost complex structure and exactly two fixed points. The homology groups of these manifolds may have arbitrary large rank in any intermediate dimension. The proof relies on fiberwise connected sum construction ([3]) and triviality of obstruction homotopy groups ([1]).

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