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A natural sequence, ZeFiRoD with all algebraic numbers, and e, π, log 2, etc.

Autogenerated sequences of numbers, functions and functionals

Graph $g_i \supseteq \{(g_j, g_k)\}$
Input g_j ⟶ Output $g_k = g_i(g_j)$

An algorithm of autogeneration

Define the genome of the generator, i.e. the first elements $(g_1, g_2, \ldots, g_S)$; e.g. for the «Primes» sequence: (NextPrime, 2). Then repeat the loop:

- **Assemble** the next compatible type pair (g_i, g_j): the multifunction (or multi-valued map) g_i can be evaluated with g_j as argument
- **Evaluate** $g_i(g_j)$ which is a value or a set of values ; e.g. : NextPrime(2) = 3 — «Eval» stems from the question: ⟶ ?
- **Parameterize** this set if it is infinite (or sometimes, reject it, if the rules are so) **or**
- **Extract** the values if the set is finite, using a predefined height rule (e.g. beginning with the object of the smallest norm and then of smallest argument)
- **Calculate norm, real and imaginary parts** if the extracted object is a complex number. This step may not exist.

«Assemble» may be symbolized by: $g_j \overset{?}{\longrightarrow} g_k$

Zφϱ∂ : A universal and wonderful mathematical Object !

New !

Generation of the ZeFiRoD sequence (alias Zφϱ∂)

Rules here: « Only 1 variable Z and 2 parameters A_1 and A_2 », « Instantiation of parameters first, and only with numbers », etc.

First elements: Except the differentiation, they all come from the 3 possible remaining questions around arrows:

Z = Polynomial function Z in ℂ = Identity =: g_1 — ? ↻

φ = Fixed points of a function (or of a multivalued function) = {z, (z,z) ∈ graph of the multifunction} =: g_2 — ↻ ?

φ' = Fixed points of a multifunctional =: g_3

ϱ = Reverse functional (transforms a graph $\{(z_s, z_t), s,t \in \ldots\}$ into $\{(z_t, z_s), s,t \in \ldots\}$) =: g_4 — ? ⟶

ϱ' = Reverse map applied to a multifunctional =: g_5 It exchanges inputs and outputs. — or ? ⇆

∂ = Multidifferentiation =: g_6 In the ZφϱΔ sequence, ∂ is replaced by Δ: $(\Delta f)(z) = f(z+1) - f(z)$

How soon, in Zφϱ∂, do unexplained properties between universal constants occur (in comparison with the expected probabilities) ?

Generation of the Zφϱ∂ sequence GenSeqPL20120702-ZeFiRoD1:

We do not assemble g_1 with g_1 , nor later, g_2 with g_2 , g_3 with g_1 , etc. because types are not compatible. We assemble g_2 with g_1 and evaluate:

$g_2(g_1) = \varphi(Z)$ = Set of the fixed points of the identity function Z = ℂ whose cardinal is not finite ; So, here we generate the parameter whose possible values are in ℂ: $A = g_7$

$g_4(g_1) = \varrho(Z)$ = Reverse function of Z = Z = Clone (It means: previously generated in the sequence). We go on, assembling only compatible type objects:

$g_3(g_4) = \varphi'(\varrho)$ = {Fixed points of ϱ} = {Multifunctions that do not change when their graph is reversed}: This infinite set of functions is not parameterized (ZeFiRoD rule)

$g_5(g_4) = \varrho'(\varrho)$ = Reversed graph of ϱ = Graph of ϱ = ϱ = Clone. The next clones will be ignored.

$g_6(g_1) = \partial(Z)$ = dz/dz = derivative of the polynomial function Z =: $\underline{1}$ = g_8 = Constant function whose image is {1}

$g_3(g_6) = \varphi'(\partial)$ = An infinite set of exponential functions that we parameterize: $Ae^Z = g_9$ — ∂ ↻ ? — $\partial(Ae^Z) = Ae^Z$

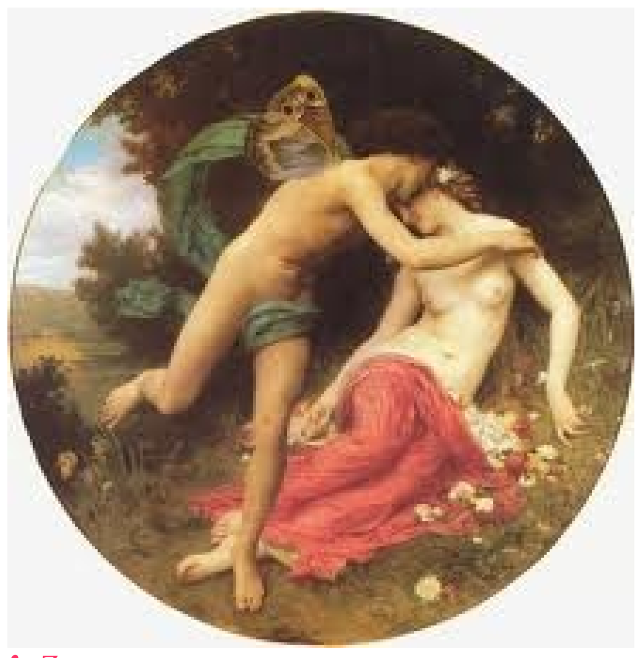
$g_5(g_6) = \varrho'(\partial)$ = $\int_{A_1}^{Z} \cdot + A_2 = g_{10}$ = Integration = Inverse of differentiation

$g_2(g_8) = \varphi(\underline{1}) = 1 = g_{11}$ — $\underline{1}$ ↻ ? — : The only fixed point of the constant function $\underline{1}$ is the number 1

$g_4(g_8) = \varrho(\underline{1})$ = Graph 1×ℂ : This multifunction whose domain {1} has only one element, is not accepted (ZeFiRoD rule)

$g_6(g_8) = \partial(\underline{1}) = d\underline{1}/dz = \underline{0} = g_{12}$ = The zero function

Numbers(Zφϱ∂) is a denumerable algebraically closed transcendental extension of the field $\overline{\mathbb{Q}}$ in ℂ



Flora and Zephyr

William Bouguereau (1875)

$g_9(g_{11}) = Ae^Z$ (with $A = g_{11} = 1$) = $e^Z = g_{13}$

$g_{10}(g_{11}) = \{ \int_{A_1}^{Z} \cdot + A_2$ (with $A_1 = 1$) = $\int_1^Z \cdot + A = g_{14}$, $\int_1^Z \cdot + A$ (with A=1) = $\int_1^Z \cdot + 1 = g_{15}$, $\int_{A_1}^{Z} \cdot + A_2$ (with $A_2 = 1$) = $\int_A^Z \cdot + 1 = g_{16}$ }

$g_2(g_{12}) = \varphi(\underline{0}) = 0 = g_{17}$ — $\underline{0}$ ↻ ? — 0 is the fixed point of the zero map

Zero, zefiro, zephyrus, zephyr, cypher, sifr, chiffre, …

$g_2(g_{13}) = \varphi(e^Z)$ = The infinite set of the fixed points of e^Z. We parameterize it with a positive integer N: a sequence with increasing norms: $\varphi(e^Z)_N = g_{18}$ — e^Z ↻ ?

$g_4(g_{13}) = \varrho(e^Z)$ = Logarithms ± 2π i k. Defining log such that $-\pi < \Im(\log(Z)) \leqslant \pi$, we parameterize with increasing norms: $\log(Z) + 2\pi i((-1)^N(2N-1)+1)/4 = g_{19}$

$g_{13}(g_{11}) = e^Z$ (with $Z = g_{11} = 1$) = $e^1 = e = g_{20}$

Conjectures: e (or later in Zφϱ∂: $1/\pi, \sqrt{\pi}, e^{\pi}, e^{\pi^2}, e^{\sqrt{2}}, \ldots$) ∉ Periods, and so, Zφϱ∂ numbers ⊄ Periods

$g_{15}(g_1) = \int_1^Z \cdot + 1$ (with · = Z) = $\int_1^Z t\,dt + 1 = [t^2/2]_1^Z + 1 = Z^2/2 + 1/2 = g_{21}$ $g_3(g_{15}) = \varphi'(\int_1^Z \cdot + 1) = e^{Z-1} = g_{22}$ …

$g_{10}(g_{17})_1 = \int_{A_1}^{Z} \cdot + A_2$ (with $A_1 = 0$) = $\int_0^Z \cdot + A = g_{24}$ … $g_{14}(g_{17}) = \int_1^Z \cdot + A$ (A=0) = $\int_1^Z \cdot = g_{27}$ $g_{16}(g_{17}) = \int_A^Z \cdot + 1$ (A=0) = $\int_0^Z \cdot + 1 = g_{28}$

$g_{18}(g_{11}) = \varphi(e^Z)_N$ (with N=1) = $\varphi(e^Z)_1 = 1.37\cdots e^{1.33\cdots i} = g_{29}$ = The fixed point of e^Z with the smallest norm (and argument > 0)

Norm$(g_{29}) = |\varphi(e^Z)_1| = 1.37\cdots = g_{30}$ $\Re(g_{29}) = 0.318\cdots = g_{31}$ $\Im(g_{29}) = 1.33\cdots = g_{32}$

$g_{19}(g_{11}) = \log(Z) + 2\pi i((-1)^N(2N-1)+1)/4$ (with N=1) = $\log(Z) = g_{33}$ … $g_4(g_{21}) = \varrho(Z^2/2 + 1/2) = \mathrm{Roots}_T(T^2/2 + 1/2 - Z) = \pm(2Z-1)^{1/2} = g_{41}$ …

$g_{27}(g_8) = \int_1^Z \cdot$ (with · = $\underline{1}$) = $\int_1^Z 1\,dt = [t]_1^Z = Z-1 = g_{63}$ … $g_{28}(g_8) = \int_0^Z \cdot + 1$ (with · = $\underline{1}$) = $\int_0^Z 1\,dt + 1 = [t]_0^Z + 1 = Z+1 = g_{68}$ …

Which numbers or functions are in Zφϱ∂, and which are not ?

The first numbers are: 1, 0, e, $1.37\cdots e^{1.33\cdots i}$, 1.37···, 0.318···, 1.33···, $e^e = 15.15\cdots$, 1/2, $(e^2+1)/2$, 1/e, … , 2π i, …

Many integer sequences can be extracted: Integers, Numerators and Denominators of the rational numbers, Floor, Ceiling or Round of the norms, …

Integer sequences can be produced from each positive number of Zφϱ∂: Bits, Digits in any base, Terms of the continued fraction, Numerators and Denominators of the successive convergents, …

Any algebraic number is in ZeFiRoD: $\overline{\mathbb{Q}} \subset$ Zφϱ∂

0, Z-1 and Z+1 ∈ Zφϱ∂ ⇒ ℤ ⊂ Zφϱ∂ and more: ℚ ⊂ Zφϱ∂ because:

If the numbers z_1 and z_2 ∈ Zφϱ∂, we have -1, log(Z) and Ae^Z ∈ Zφϱ∂ ⇒ $z_1 e^{\log(z_2)} = z_1 z_2$ ∈ Zφϱ∂ and then $-\log(z_2)$ and $z_1 e^{-\log(z_2)} = z_1/z_2$ ∈ Zφϱ∂.

Let us show now that any rational polynomial function is in ZeFiRoD: $\int_0^Z \cdot + A$ and $\underline{0}$ ∈ Zφϱ∂ ⇒ $\int_0^Z 0\,dt + a_0 = a_0 \underline{1}$ ∈ Zφϱ∂ for any rational number a_0.

Then, $\int_0^Z a_0 \underline{1}dt + a_1 = a_0 Z + a_1$ ∈ Zφϱ∂ for any a_0 and a_1 ∈ ℚ. Then, $\int_0^Z (a_0 t + a_1)dt + a_2 = a_0 Z^2/2 + a_1 Z + a_2$ ∈ Zφϱ∂ for any rational numbers a_0, a_1, a_2 ; etc.

So, any polynomial function with rational coefficients is generated sooner or later in ZeFiRoD, and when its reverse is evaluated at 0, the roots are extracted.

π is generated too, because g_{19} evaluated at N = 2 produces $\log(Z) + 2\pi i$ which gives 2π i at Z = 1. Then, 2π i divided by the algebraic number 2i gives π.

References: //oeis.org/ *The On-Line Encyclopedia of Integer Sequences™* //www.experimentalmath.info/ *Experimental Mathematics Website*