

On the Generalized Wintgen Inequality

Ion Mihai
University of Bucharest

imihai@fmi.unibuc.ro

Abstract

In 1979, Wintgen proved that the Gauss curvature K , the squared mean curvature $\|H\|^2$ and the normal curvature K^\perp of any surface M^2 in \mathbf{E}^4 always satisfy the inequality $K \leq \|H\|^2 - |K^\perp|$.

The normal scalar curvature conjecture, also known as the DDVV conjecture, was formulated by De Smet, Dillen, Verstraelen and Vrancken in 1999. Let M^n be a submanifold of a real space form $\tilde{M}^{n+m}(c)$. Then

$$\rho \leq \|H\|^2 - \rho^\perp + c,$$

where ρ is the normalised scalar curvature and ρ^\perp is the normalised normal scalar curvature.

It was proven recently by Lu and by Ge and Tang independently.

We obtained the DDVV inequality, also known as generalized Wintgen inequality, for Lagrangian submanifolds in complex space forms.

We investigate Lagrangian submanifolds which satisfy identically the equality case. Such submanifolds are Chen submanifolds.

Also we state a DDVV inequality for slant submanifolds in complex space forms.

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