On the Heyde characterization theorem for discrete Abelian groups

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Abstract

In 1970 Heyde proved the following theorem.

Heyde theorem. *Let ξ_j, $j = 1, 2, ..., n$, $n \geq 2$, be independent random variables. Let α_j, β_j be nonzero constants such that $\beta_i\alpha_i^{-1} \pm \beta_j\alpha_j^{-1} \neq 0$ for all $i \neq j$. If the conditional distribution of $L_2 = \beta_1\xi_1 + \cdots + \beta_n\xi_n$ given $L_1 = \alpha_1\xi_1 + \cdots + \alpha_n\xi_n$ is symmetric then all random variables ξ_j are Gaussian.*

Formulate the following problem.

Problem 1. *Let X be a locally compact second countable Abelian group, $\mathrm{Aut}(X)$ the set of topological automorphisms of X. Let ξ_j, $j = 1, 2, ..., n$, $n \geq 2$, be independent random variables with values in X and distributions μ_j. Consider the linear forms $L_1 = \alpha_1\xi_1 + \cdots + \alpha_n\xi_n$ and $L_2 = \beta_1\xi_1 + \cdots + \beta_n\xi_n$, where $\alpha_j, \beta_j \in \mathrm{Aut}(X)$ such that $\beta_i\alpha_i^{-1} \pm \beta_j\alpha_j^{-1} \in \mathrm{Aut}(X)$ for all $i \neq j$. Assume that the conditional distribution of the linear form L_2 given L_1 is symmetric. Describe possible distributions μ_j.*

We solve Problem 1 in the class of countable discrete Abelian groups.

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