INITIAL VALUE PROBLEMS FOR FIRST ORDER DIFFERENTIAL SYSTEMS

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Abstract

The purpose of the present paper is to study the existence of solutions to initial value problems for nonlinear first order differential systems with nonlocal conditions, of the type

$$\begin{cases} x'(t) = f_1(t, x(t), y(t)) \\ y'(t) = f_2(t, x(t), y(t)) \qquad \text{(a.e. on } [0,1]) \\ x(0) = \alpha[x] \\ y(0) = \beta[y], \end{cases}$$

where, $f_1, f_2 : [0,1] \times \mathbf{R}^2 \to \mathbf{R}$ are Carathéodory functions, $\alpha, \beta : C[0,1] \to \mathbf{R}$ are linear and continuous functionals.

The fixed point principles of Perov, Schauder and Leray-Schauder are applied to a nonlinear integral operator splitted into two parts, one of Fredholm type and an another one of Volterra type. The novelty in this paper is that this approach is combined with the technique that uses convergent to zero matrices and vector norms.

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