Stochastic calculus with respect to the fractional Brownian motion

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Abstract

The fractional Brownian motion (fBm) is a centered Gaussian stochastic process $\{B_t^H, t \geq 0\}$ which is self-similar and it has stationary increments and variance $E((B_t^H)^2) = t^{2H}$, where $H \in (0,1)$ is called the Hurst parameter. The purpose of this talk is to describe some properties of the fBm, and to discuss some recent advances on the stochastic calculus with respect to this process.

The fBm and its multiparameter extension called the fractional Brownian sheet, are suitable input noises in ordinary and partial differential equations. We will present a version of the Feynman-Kac formula for the heat equation on $\mathbb{R}^d$ driven by a multiplicative fractional Brownian sheet. This formula holds it the Hurst parameters of the noise in time H_0 and space $(H_1, \ldots, H_d)$ satisfy $2H_0 + \sum_{i=1}^d H_i > d + 1$, and involves stochastic integrals of distributions with respect to the fractional Brownian sheet.

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