Asymptotic soliton type solution to Cauchy problem for singularly perturbed Korteweg-de Vries equation with variable coefficients

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Abstract

One of the fundamental equations of modern theoretical and mathematical physics is Korteweg-de Vries equation

$$u_t + 6uu_x + u_{xxx} = 0 \tag{1}$$

proposed by D. Korteweg and G. de Vries for mathematical describing solitary wave firstly discovered by Scotland scientist J.Scott-Russel in 1834. Later in the XX-th century M.Kruskal and N.Zabusky mathematically justify some physical properties of solitary wave discovered by J.Scott-Russel and proposed term "soliton". It was demonstrated that Korteweg-de Vries equation describes many interesting phenomena in plasma, waves in unharmonious lattices and others.

Mathematical models based on singularly perturbed Korteweg-de Vries equation with variable coefficients describe a number of wave processes in medium with small dispersion. Therefore problem of studying solutions to the equation is of a great importance.

We apply perturbation theory for studying asymptotic one phase soliton type solution to Cauchy problem for singularly perturbed Korteweg-de Vries equation with variable coefficients of the following form

$$\varepsilon^2 u_{xxx} = a(x,\varepsilon)u_t + b(x,\varepsilon)uu_x,$$

$$u(x,0,\varepsilon) = f\left(\frac{x}{\varepsilon}\right),$$

where functions $a(x,\varepsilon)$, $b(x,\varepsilon)$ are represented as

$$a(x,\varepsilon) = \sum_{k=0}^{\infty} a_k(x)\varepsilon^k, \quad b(x,\varepsilon) = \sum_{k=0}^{\infty} b_k(x)\varepsilon^k,$$

$a_k(x)$, $b_k(x) \in C^{(\infty)}(\mathbf{R})$, $k \geq 0$; $t \in [0;T]$; $\varepsilon > 0$ is a small parameter; function $f(\eta)$ belongs to space of quickly decreasing functions.

1. Samoylenko V., Samoylenko Yu. On Cauchy problem for Korteweg-de Vries equation with variable coefficients and small parameter. Collection of papers "Computer Algebra Systems in Teaching and Research. Differential Equations, Dynamical Systems and Celestial Mechanics", Siedlce (Poland) (2011) 128 – 135

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