Renormalized energy, Abrikosov lattice, and log gases

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Abstract

In superconductivity, one observes in certain regimes the emergence of densely packed point vortices forming perfect hexagonal lattice patterns. These are named "Abrikosov lattices" in physics. In joint work with Etienne Sandier, we showed how the distribution of these vortices is governed by a Coulomb type of interaction, which can be computed via a "Coulombian renormalized energy" which we introduced and derived rigorously from the Ginzburg–Landau model of superconductivity. Such an interaction turns out to be common in two-dimensional systems. We showed it arises in particular in the statistical mechanics of the "Coulomb gas", which contains as a specific case the Ginibre ensemble of random matrices. We also defined a one-dimensional log-interaction analogue, arising naturally in the statistical mechanics of "log gases", which contains as a specific case the so-called GUE ensemble of random matrices.

In this talk I will present the renormalized energy, examine the question of its minimization and its link with the Abrikosov lattice and weighted Fekete points. I will describe its relation with the statistical mechanics models mentioned above and show how it leads to expecting crystallisation in the low temperature limit.

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