Canonical Matrices for Systems of Forms and Linear Mappings

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Theorem 1. ([1]) (a) *For each bilinear form on a complex vector space, there is a basis in which its matrix is a direct sum, determined uniquely up to permutation of summands, of matrices of the form:*

$$H_n(\lambda) := \begin{bmatrix} 0 & I_n \\ J_n(\lambda) & 0 \end{bmatrix}, \quad \Gamma_n := \begin{bmatrix} 0 & & & & \iddots \\ & & & -1 & \iddots \\ & & 1 & 1 & \\ & -1 & -1 & & \\ 1 & 1 & & & 0 \end{bmatrix} \ (n\text{-}by\text{-}n), \quad J_n(0),$$

in which $\lambda \neq 0, (-1)^{n+1}$ *and* λ *is determined up to replacement by* λ^{-1}.

(b) *For each sesquilinear form on a complex vector space, there is a basis in which its matrix is a direct sum, determined uniquely up to permutation of summands, of matrices of the form:*

$$H_n(\lambda), \qquad \mu\Gamma_n, \qquad J_n(0), \tag{1}$$

in which $|\lambda| > 1$ *and* $|\mu| = 1$.

The proof is based on the method of reducing the problem of classifying systems of forms and linear mappings to the problem of classifying systems of linear mappings that was developed in [2] (see also [3]).

Theorem 2. ([2]) *Each system of linear mappings and bilinear/sesquilinear forms on vector spaces over* $\mathbb{R}$, $\mathbb{C}$, *and* $\mathbb{H}$ *decomposes into a direct sum of indecomposable systems uniquely up to isomorphism of summands.*

Theorem 3. ([3]) *The set of dimensions of indecomposable systems of linear mappings and bilinear/sesquilinear forms on vector spaces over* $\mathbb{C}$ *coincides with the set of positive roots defined by V.G. Kac.*

References

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[2] V.V. Sergeichuk, Classification problems for system of forms and linear mappings, *Math. USSR-Izv.* **31** (3) (1988) 481–501.

[3] V.V. Sergeichuk, Linearization method in classification problems of linear algebra, *São Paulo J. Math. Sci.* **1** (2) (2007) 219–240.