A CLASSIFICATION OF EMBEDDINGS OF NON-SIMPLY CONNECTED 4-MANIFOLDS IN 7-SPACE

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Let N be a closed connected orientable 4-manifold with torsion free integral homology. The main result is *a complete readily calculable classification of embeddings* $N \to \mathbb{R}^7$, in the smooth and in the piecewise-linear (PL) categories. Such a classification was earlier known only for simply-connected N, in the PL case by Boéchat-Haefilger-Hudson 1970, in the smooth case by the authors 2008 (arxiv:math/0808.1795). In particular, for $N = S^1 \times S^3$ we define geometrically a 1–1 correspondence between the set of PL isotopy classes of PL embeddings $S^1 \times S^3 \to \mathbb{R}^7$ and the quotient set of $\mathbb{Z} \oplus \mathbb{Z}_6$ up to equivalence $(l, b) \sim (l, b')$ for $b \equiv b' \mod 2GCD(3, l)$. This particular case allows us to disprove the conjecture on the completeness of the Multiple Haefliger-Wu invariant, as well as the Melikhov informal conjecture on the existence of a geometrically defined group structure on the set of PL isotopy classes of PL embeddings in codimension 3. For $N = S^1 \times S^3$ and the smooth case we identify the isotopy classes of embeddings with an explicitly defined quotient of $\mathbb{Z}_{12} \oplus \mathbb{Z} \oplus \mathbb{Z}$.