Geometry of Stochastic Processes

Michel Talagrand spinglasses@gmail.net
University of Paris VI and CNRS

Abstract

Given a stochastic process, that is a (finite) collection $(X_t)_{t\in T}$ of random variables, it is a fundamental problem to find upper and lower bounds for the "size of this process", as measured e.g. by the quantity $\mathsf{E}\sup_{s,t\in T}|X_s - X_t|$. Recent progress in this area has allowed the final solution of two old problems (concerning convergence of random Fourier series and of orthogonal series). We consider in particular the fundamental case of "random series of functions", where the random variables X_t are of the type $X_t = \sum_{i\le N} t_i\xi_i$, where $(\xi_i)_{i\le N}$ are independent random variables and where $t = (t_i)_{i\le N}$ are coefficients. The central question here is to relate the size of process with the "geometry of the index set". The most important situation is where the variables ξ_i are centered standard Gaussian random variables. It is understood since 1985 that in this case the best possible upper bounds are given by a suitable version of Kolmogorov's chaining. Based on this situation, we propose a long term research program in the form of a daring series of conjectures which would cover other cases of fundamental importance, and first of all the case of coin-flipping random variables (i.e. $\mathsf{P}(\xi_i = \pm 1) = 1/2$). These conjectures are precise formulations of the optimistic idea that there should be no other methods to bound such families of random variables from above than "trivial methods" (such as the use of chaining, of linearity, of the triangle inequality) and mixtures of these.

AMS Classification: 60G99.